# 裂隙含水层中地下水非线性井流问题研究

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[摘　要] 【目的】探讨裂隙含水层中地下水井流问题的精确计算方法,为裂隙含水层中地下水渗流的精确计算提供理论支持。【方法】建立了裂隙含水层中的地下水井流模型,就具有普遍性的 Forchheimer 非线性渗流规律,通过 Boltzmann 变换,对非线性渗流模型进行了求解。【结果】从理论上求解出了裂隙含水层中非线性渗流区域内渗流速度和水头降深的解析表达式,并通过引入无量纲时间、紊流因子和井流函数,对所求解析解进行了化简。【结论】揭示了裂隙含水层中非线性流的客观存在,建立了非线性渗流模型并进行了求解。

[关键词] 裂隙含水层;非线性流;渗流速度;水头降深

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# Study on the problem of the nonlinear well flow in the fractured rocks

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**Abstract**:【Objective】The study discussed more accurate calculation method of the problem of well flow in the aquifer layer.【Method】The more universal Forchheimer non-linear flow law was used in the text, the water balance equation was established and solved in the nonlinear region by Boltzmann transform.【Result】The analytical expression of seepage velocity and drawdown was given. By the introduction of non-dimensional time and turbulence factor and well function, the solutions were simplified.【Conclusion】The study reveals that the non-linear flow exists objectively in the fractured rocks, and the seepage equation is established and solved. The study improves and perfects the theory of flow and provides support to the well flow calculation in theory.

**Key words**: fractured rock; nonlinear flow; specific discharge; drawdown

在裂隙含水层中,当含水层厚度较小、介质粒径较大或者渗流速度较大时,水力梯度不再是渗流速度的线性函数,并且在裂隙附近渗流区域出现了非线性流,因而达西定律不再适于裂隙含水层中地下水渗流问题的计算。土耳其学者 Sen[1-3] 就充分紊流(即非线性指数取 2 时)的情况,对裂隙含水层中的非线性井流问题进行了研究,求出了渗流速度和水头降深的解析表达式。但是在实际渗流问题中,非线性指数和渗流系数均随着介质和具体渗流条件的变化而变化,并不是常量,可见 Sen[1-3] 的研究结果仍具有一定的片面性和局限性。为了更准确、客观地描述和解决裂隙含水层中的地下水非线性井流问题,进一步丰富和完善地下水非线性渗流理论,又便于数学上的推导,本研究选用更具有普遍性的 Forchheimer 非线性渗流规律 $J=av(x,t)+bv(x,t)^n$,对裂隙含水层井流问题的渗流模型进行了求解,并对渗流速度和水头降深的解析表达式进行了推导,以期为裂隙含水层中非线性井流问题的精确

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计算提供理论支持。

# 1 裂隙含水层水量平衡方程的建立

如图 1 所示,在厚度为 $m$ 的广阔含水层中,有 1 条垂直于含水层的平面裂隙,其长度为 $L$,含水层贮水系数为 $S$,水力传导系数为 $T$。为了便于数学描述,对裂隙含水层作如下假设:(1)含水层均质各向同性,上下边界为隔水顶板和底板;(2)裂隙垂直穿透整个含水层,相对于长度其宽度很小,其中的储水量可以忽略不计;(3)水流为垂直于裂隙的层流;(4)在抽水过程中,整个裂隙中各处的水位相同;(5)定流量抽水,即在整个抽水过程中流量 $Q$ 为常数。

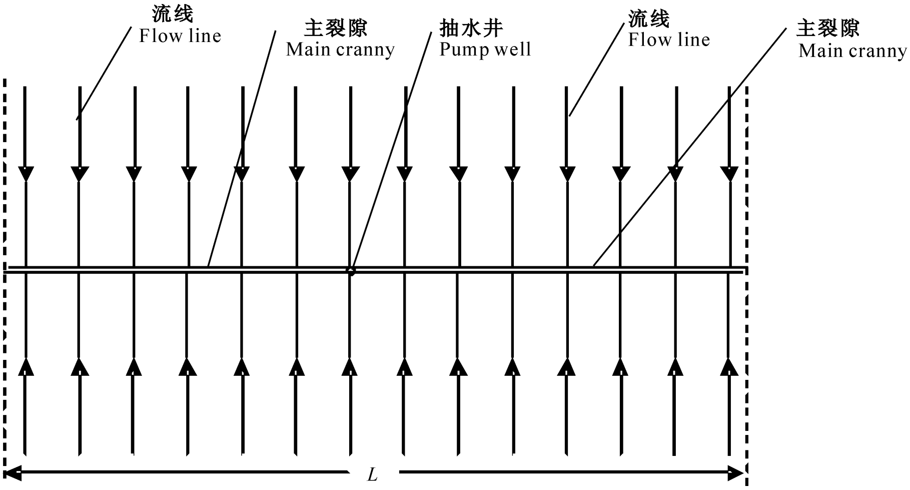


图 1 裂隙含水层示意图

Fig. 1 Figure of water carrier in cranny

考虑两边到裂隙的垂直距离为 $x$,在宽度为的 $\Delta x$ 的区域,在 $\Delta t$ 时间段内,有如下水量平衡方程[1,4-11]:

$$\Delta t[Q(x,t)-Q(x+\Delta x,t)]=2SL\Delta x[h(x,t)-h(x,t+\Delta t)]。\tag{1}$$

式中:$Q(x,t)$和 $h(x,t)$分别为含水层中与裂隙垂直距离为 $x$ 断面上 $t$ 时刻的流量及水头,$S$ 为含水层的储水系数,$L$ 为裂隙宽度。

当 $\Delta x\to 0$ 且 $\Delta t\to 0$ 时,方程(1)变为 :

$$\frac{\partial Q(x,t)}{\partial x}=2SL\frac{\partial h(x,t)}{\partial t}。\tag{2}$$

而流量 $Q(x,t)$又可以表示为:

$$Q(x,t)=2SLmv(x,t)。\tag{3}$$

式中:$m$ 为含水层厚度,$v(x,t)$为渗流速度。

将(3)式带入(2)可得:

$$\frac{\partial v(x,t)}{\partial x}=\frac{S}{m}\frac{\partial h(x,t)}{\partial t}。\tag{4}$$

(4)式也可以写成:

$$\frac{\partial v(x,t)}{\partial x}=-\frac{S}{m}\frac{\partial s(x,t)}{\partial t}。\tag{5}$$

式中:$s(x,t)$为水头降深。

边界条件和初始条件为:

$$\lim_{x\to 0}[2Lmv(x,t)]=Q,\tag{6}$$

$$\lim_{x\to\infty}[s(x,t)]=0,\tag{7}$$

$$\lim_{t\to 0}[s(x,t)]=0。\tag{8}$$

# 2 裂隙含水层中非线性井流方程的建立与求解

## 2.1 地下水非线性渗流方程的建立

在裂隙含水层中,Forchheimer 非线性渗流规律可表示为:

$$-\frac{\partial s(x,t)}{\partial x}=av(x,t)+bv(x,t)^n。\tag{9}$$

式中:$a$、$b$ 为常数,特别地,当 $b=0$ 时,上面的渗流规律即为达西定律,渗流系数 $K=1/a$;$n$ 为非线性指数,由水力学可知 $1\leqslant n\leqslant 2$,只有在充分紊流的情况下 $n=2$[12-16]。

引入 Boltzmann 变换,可得:

$$\eta=\frac{x^2}{4t}。\tag{10}$$

由此可得:

$$\frac{\partial s(x,t)}{\partial x}=\frac{x}{2t}\frac{\mathrm{d}s(\eta)}{\mathrm{d}\eta},\tag{11}$$

$$\frac{\partial v(x,t)}{\partial x}=\frac{x}{2t}\frac{\mathrm{d}v(\eta)}{\mathrm{d}\eta},\tag{12}$$

$$\frac{\partial s(x,t)}{\partial t}=-\frac{x}{4t^2}\frac{\mathrm{d}s(\eta)}{\mathrm{d}\eta}。\tag{13}$$

由(9)式和(11)式得:

$$\frac{\mathrm{d}s(\eta)}{\mathrm{d}(\eta)}=-\frac{2t}{x}[av(\eta)+bv(\eta)^n]。\tag{14}$$

由(5)式和(12)、(13)式得:

$$\frac{\mathrm{d}v(\eta)}{\mathrm{d}\eta}=-\frac{S}{m}\frac{x}{2t}\frac{\mathrm{d}s(\eta)}{\mathrm{d}\eta}。\tag{15}$$

由(14)、(15)式可得:

$$\frac{\mathrm{d}v(\eta)}{\mathrm{d}\eta}+\frac{aS}{m}v(\eta)+\frac{bS}{m}v(\eta)^n=0。\tag{16}$$

该方程即是关于渗流速度的地下水非线性渗流方程。

在 Boltzmann 变换下,其初始条件和边界条件也相应地变为:

$$\lim_{\eta\to 0}[2Lmv(\eta)]=Q,\tag{17}$$

$$\lim_{\eta\to\infty}[s(\eta)]=0。\tag{18}$$

## 2.2　非线性渗流方程的求解

方程(16)是 Bernoulli 方程,令

$$\xi(\eta)=v(\eta)^{1-n}。\tag{19}$$

可得:

$$\frac{\mathrm{d}\xi(\eta)}{\mathrm{d}\eta}+(1-n)\frac{aS}{m}\xi(\eta)+(1-n)\frac{bS}{m}=0。\tag{20}$$

解得:

$$\xi(\eta)=c\mathrm{e}^{\frac{(n-1)aS}{m}\eta}-\frac{b}{a}。\tag{21}$$

式中:$c$ 为积分常量。

所以

$$v(\eta)=\mathrm{e}^{-\frac{aS}{m}\eta}\left[c-\frac{b}{a}\mathrm{e}^{-\frac{(n-1)aS}{m}\eta}\right]^{-\frac{1}{n-1}}。\tag{22}$$

由边界条件(17)式得:

$$c=(\frac{Q}{2Lm})^{1-n}+\frac{b}{a}。\tag{23}$$

代入(22)式可得渗流速度为:

$$v(\eta)=\frac{Q}{2Lm}\mathrm{e}^{-\frac{aS}{m}\eta}\left[1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}(1-\mathrm{e}^{-\frac{(n-1)aS}{m}\eta})\right]^{-\frac{1}{n-1}}。\tag{24}$$

又因为:

$$\frac{\mathrm{d}v(\eta)}{\mathrm{d}\eta}=\frac{aSQ}{2Lm^2}\mathrm{e}^{-\frac{aS}{m}\eta}\left(1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}(1-\mathrm{e}^{-\frac{(n-1)aS}{m}\eta})\right)^{-\frac{1}{n-1}}\times$$
$$\left[1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}\mathrm{e}^{-\frac{(n-1)aS}{m}\eta}\left(1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}(1-\mathrm{e}^{-\frac{(n-1)aS}{m}\eta})\right)^{-1}\right]。\tag{25}$$

将(25)式代入(15)式可得:

$$\frac{\mathrm{d}v(\eta)}{\mathrm{d}\eta}=\frac{aQt}{Lmx}\mathrm{e}^{-\frac{aS}{m}\eta}\left(1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}(1-\mathrm{e}^{-\frac{(n-1)aS}{m}\eta})\right)^{-\frac{1}{n-1}}\times$$
$$\left[1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}\mathrm{e}^{-\frac{(n-1)aS}{m}\eta}\left(1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}(1-\mathrm{e}^{-\frac{(n-1)aS}{m}\eta})\right)^{-1}\right]。\tag{26}$$

由(11)式可得水力梯度为:

$$\frac{\partial s(x,t)}{\partial x}=-\frac{aQ}{2Lm}\mathrm{e}^{-\frac{aSx^2}{4mt}}\left(1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}(1-\mathrm{e}^{-\frac{(n-1)aSx^2}{4mt}})\right)^{-\frac{1}{n-1}}\times$$
$$\left[1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}\mathrm{e}^{-\frac{(n-1)aSx^2}{4mt}}\left(1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}(1-\mathrm{e}^{-\frac{(n-1)aSx^2}{4mt}})\right)^{-1}\right]。\tag{27}$$

两边从 $x$ 到∞积分,可得水头降深为:

$$s(x,t)=\frac{aQ}{2Lm}\int_x^\infty\mathrm{e}^{-\frac{aSx^2}{4mt}}\left(1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}(1-\mathrm{e}^{-\frac{(n-1)aSx^2}{4mt}})\right)^{-\frac{1}{n-1}}\times$$
$$\left[1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}\mathrm{e}^{-\frac{(n-1)aSx^2}{4mt}}\left(1+\frac{b}{a}(\frac{Q}{2Lm})^{n-1}(1-\mathrm{e}^{-\frac{(n-1)aSx^2}{4mt}})\right)^{-1}\right]\mathrm{d}x。\tag{28}$$

定义无量纲时间:

$$u_f=\frac{(n-1)aSx^2}{4mt},\tag{29}$$

及紊流因子:

$$B=\frac{b}{a}(\frac{Q}{2Lm})^{n-1}。\tag{30}$$

非线性井流函数为:

$$W(u_f)=\frac{2Lm}{aQx}s(x,t)。\tag{31}$$

则有:

$$W(u_f)=\frac{1}{2\sqrt{u_f}}\int_{u_f}^\infty(1+B(1-\mathrm{e}^{-u_f}))^{-\frac{1}{n-1}}\times$$
$$[1+B\mathrm{e}^{-u_f}(1+B(1-\mathrm{e}^{-u_f}))^{-1}]\frac{\mathrm{e}^{-\frac{u_f}{n-1}}}{\sqrt{u_f}}\mathrm{d}u_f。\tag{32}$$

# 3　结　论

本研究对裂隙含水层中的非线性井流问题进行

了较为准确、客观的描述,推导出了裂隙含水层非线性井流模型的解析解,丰富和完善了地下水非线性渗流理论。为了便于应用,以后应继续研究解析解的化简及其计算机实现,使研究的结果更简单、更实用。

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